PARENT OFFSPRING CORRELATIONS UNDER HALF-SIB MATING SYSTEM

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1. INTRODUCTION

The study of correlation between different relatives under different inbred systems has been done by different authors such as Fisher [1], Kempthorne [6], Horner [5], Li [10], Korde [9], George [3] and George and Narain [4]. The study of wright is through the path coefficient approach. Eventhough this method is very easy for the calculations of the correlation between relatives, the joint distribution between the two relatives cannot be obtained by this method. Kempthorne and others delt with the study of correlations in inbred population by the method of generation matrix approach. By this method they could derive the joint distribution of the pairs of relatives at any generation of a specified system of mating and thus the correlation is worked out directly from the two-way table of the relatives, known as the "Correlation Table". All these authors confine to the two systems of mating namely full-sib mating and parent off-spring mating for both autosomal as well as sex-linked gene case. As the generation matrix in the case of half-sibs are not eaisly possible, the study of the joint distribution between half-sibs paris and the correlation there-from are not attempted so far.

In this paper an attempt has been made to study the parent offspring correlation under the different generations of half-sib mating for one parent and one offspring, one parent and several offspring and both the parents and several offspring cases.

2.1 Joint Distribution And the Correlation Coefficient of Parent-Offspring Pairs Under Half-Sib Mating

The joint distribution of parent and offspring pairs under the first generation of half-sib mating can be obtained as :

$$\underline{Z}^{(1)} = \underset{\sim}{B}\, \underline{U}^{(0)}$$

Where,

$$\underset{\sim}{B}=\begin{bmatrix} 1 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 1 \end{bmatrix}$$

is the generation matrix of parent-offspring mating in the autosomal gene case as explained by George [3] and $U^{(0)}$ is the column vector of the frequencies of half-sib pairs under random mating consider the case of single locus with two alleles, say A and a, thus the frequencies of half-sib pairs under random mating arranged in the order of (AA,AA), (AA,Aa), (AA,aa), (Aa,AA), (Aa,Aa), (Aa,aa), (aa,AA), aa,Aa), (aa,aa), is a column as :

$$\underset{\sim}{U}^{(0)}=\begin{bmatrix} \frac{1}{2}\,p^3\,(1+p) \\ \frac{1}{2}\,p^2q\,(1+2p) \\ \frac{1}{2}\,p^2\,q^2 \\ \frac{1}{2}\,p^2\,q\,(1+2p) \\ \frac{1}{2}\,p\,q\,(1+4\,p\,q) \\ \frac{1}{2}\,p\,q^2\,(1+2q) \\ \frac{1}{2}\,p^2\,q^2 \\ \frac{1}{2}\,p\,q^2\,(1+2q) \\ \frac{1}{2}\,q^3\,(1+q) \end{bmatrix}$$

and $\underset{\sim}{Z}^{(1)}$ stands for the column vector of frequencies of the parent off-spring pairs in the first generation of half-sib mating. Hence the joint distribution of parent-offspring pairs in the first generation of half-sib mating can be obtained as given in table 1.

TABLE 1

Offspring

		AA	*Aa*	*aa*	*Total*
	AA	$\frac{1}{4}p^2(1+3p)$	$\frac{3}{4}p^2q$	0	p^2
parent	*Aa*	$\frac{1}{8}pq(1+6p)$	pq	$\frac{1}{8}pq(1+6q)$	$2pq$
	aa	0	$\frac{3}{4}pq^2$	$\frac{1}{4}q^2(1+3q)$	q^2
Total		$\frac{1}{8}p(1+7p)$	$\frac{7}{4}pq$	$\frac{1}{8}q(1+7q)$	1

The correlation coefficient between the parent and offspring in the first generation of half-sib mating can be obtained from the above table by assuming additive genic effect and scoring AA, Aa, aa as 2, 1, 0 respectively as :

$$\text{H. S.}\, r^{(1)}\, p\text{—}0=0.5892$$

In the same manner the column vectors of the joint distribution of parent-offspring pairs in the second, third and fourth generations of half-sib mating can be obtained as :

$$\underset{\sim}{Z}^{(2)}=\underset{\sim}{B}\ \underset{\sim}{U}^{(1)}$$

$$\underset{\sim}{Z}^{(3)}=\underset{\sim}{B}\ \underset{\sim}{U}^{(2)}$$

$$\underset{\sim}{Z}^{(4)}=\underset{\sim}{B}\ \underset{\sim}{U}^{(3)}$$

Where $\underset{\sim}{U}^{(1)}$, $\underset{\sim}{U}^{(2)}$ and $\underset{\sim}{U}^{(3)}$ are the column vectors of the frequencies of the half-sib pairs from first, second and third generations of half-sib mating, derived directly as that of the random mating case. These are as shown below:

$$\underset{\sim}{U}^{(1)}=\begin{bmatrix} \frac{p}{64}(1+11p+36p^2+16p^3) \\ \frac{pq}{16}(1+9p+8p^2) \\ \frac{pq}{16}(3+16pq) \\ \frac{pq}{16}(1+9p+8p^2) \\ \frac{pq}{16}(9+16pq) \\ \frac{pq}{16}(1+9q+8q^2) \\ \frac{q}{64}(1+11q+36q^2+16q^3) \end{bmatrix}$$

$$\underline{U}^{(2)}=\begin{bmatrix} \frac{p}{1024}\,(45+315p+560p^2+104p^3+24p^2q) \\ \frac{pq}{512}\,(59+256\,p+128\,p^2) \\ \frac{pq}{1024}\,(61+128\,p\,q) \\ \frac{pq}{512}\,(59+256\,p+128\,p^2) \\ \frac{pq}{256}\,(149+128\,p\,q) \\ \frac{pq}{512}\,(59+256\,q+128\,q^2) \\ \frac{pq}{1024}\,(61+128\,p\,q) \\ \frac{pq}{512}\,(50+256\,q+128\,q^2) \\ \frac{q}{1024}\,(45+315\,q+560\,q^2+104\,q^3+24\,pq^2) \end{bmatrix}$$

$$\underline{U}^{(3)}=\begin{bmatrix} \frac{p}{16384}\,(146+7275\,p+6624\,p^2+1024\,p^3) \\ \frac{pq}{8192}\,(1251+3312\,p+1024\,p^2) \\ \frac{pq}{16384}\,(1029+1024\,pq) \\ \frac{pq}{8192}\,(1251+3312\,p+1024\,p^2) \\ \frac{pq}{8192}\,(4554+2048\,pq) \\ \frac{pq}{8192}\,(12151+3312\,q+1024\,q^2) \\ \frac{pq}{16384}\,(1029+1024\,pq) \\ \frac{pq}{8192}\,(1215+3312\,q+1024\,q^2) \\ \frac{q}{16384}\,(1461+7275\,q+6624\,q^2+1024\,q^3) \end{bmatrix}$$

The correlations between the parent and off spring in second, third and fourth generations of half-sib mating are calculated from correlation tables formed from $\underline{Z}^{(2)}$ $\underline{Z}^{(3)}$ and $\underline{Z}^{(4)}$ as

$$H.S r^{(2)}\ p-0 \quad = 0.6672$$

$$H.S r^{(3)}\ p-0 \quad = 0.7248$$

$$H.S r^{(4)}\ p-0 \quad 0.7697$$

2.2 Correlation Between Parent and K Off-spring

Consider a single locus with two alleles *A-a*, so that with additive geneic effect we score the values of the genotypes *AA*, *Aa aa* as 2, 1, 0 respectively. Again consider that there are *K* off-spring produced from the matings of these genotypes. The scores of these resulting off-spring is then obtained by adding the scores of the individual off-spring. It is then obvious that the maximum score for *K* off-spring is 2 *k*, when all the off-spring are of the type *AA*. Similarly the minimum score of ten off-sprieg will be '0', when all the offspring are of the type *aa*. So far as the scores of parents are concerned, we have two distinct case. In one we consider that the sum of scores of two parents which will range between 0 and 4 and in the second case we consider only the score of one parent, ranging between 0 and 2.

The theoretical procedure for finding the correlations between both the parents and K offspring and the correlation between one parent and K offspring are exactly same as that of the autosomal gene case for parent-offspring mating and full-sib mating explained by George and Narain [4]. The correlation between both the parent and K offspring and that of one parent and K offspring were worked out in the lines of George and Narain [4], for the first four generations of half-sib mating, and they are as given in tables 2 and 3.

TABLE 2

Correlation between both the parents and K offspring in the first four generations of half-sib mating.

Generation	0	1	2	3	4
Corre : Coeff	$\sqrt{\frac{K}{1+K}}$	$\sqrt{\frac{5K}{4+5K}}$	$\sqrt{\frac{25K}{14+25K}}$	$\sqrt{\frac{117K}{50+117K}}$	$\sqrt{\frac{529K}{178+529K}}$

Note : 0^{th} generation stands for random mating

TABLE 3

Correlation between one parent and K offspring in the first four generations of half-sib mating.

Generation,	0	1	2	3	4
Corre : Coeff	$\sqrt{\frac{K}{2K(1-K}}$	$\sqrt{\frac{5K}{8K(4+5K)}}$	$\sqrt{\frac{25K}{56K(14+25K)}}$	$\sqrt{\frac{117K}{156K(50+117K}}$	$\sqrt{\frac{529K}{668K(178+529K)}}$

Note : Oth generation stand for random mating.

By giving different values to K from 1 to 10, the correlation between one parent and K offspring in the first four generations of half-sib mating were calculated as given in tables 4 and 5.

The trend of the correlation in the different generations, in the case of both the parents and K offspring, as well as one parent and K offspring were represented in figures 1 and 2 respectively for varying number of offspring on page 42.

TABLE 4

Correlation between both the parents and K offspring when K= 1 to 10, for the first four generations of half-sib mating

Generation / *No. of offspring*	*0*	*1*	*2*	*3*	*4*
1.	0.7071	0.7453	0.8006	0.8370	0.8650
2.	0.8165	0.8451	0.8838	0.9077	0.9252
3.	0.8660	0.8885	0.9179	0.9355	0.9482
4.	0.8944	0.9128	0.9365	0.9505	0.9604
5.	0.9128	0.9284	0.9483	0.9598	0.9674
6.	0.9258	0.9393	0.9563	0.9661	0.9730
7.	0.9354	0.9473	0.9622	0.9708	0.9768
8.	0.9428	0.9534	0.9667	0.9743	0.9796
9.	0.9486	0.9583	0.9702	0.9770	0.9818
10.	0.9534	0.9622	0.9731	0.9792	0.9835

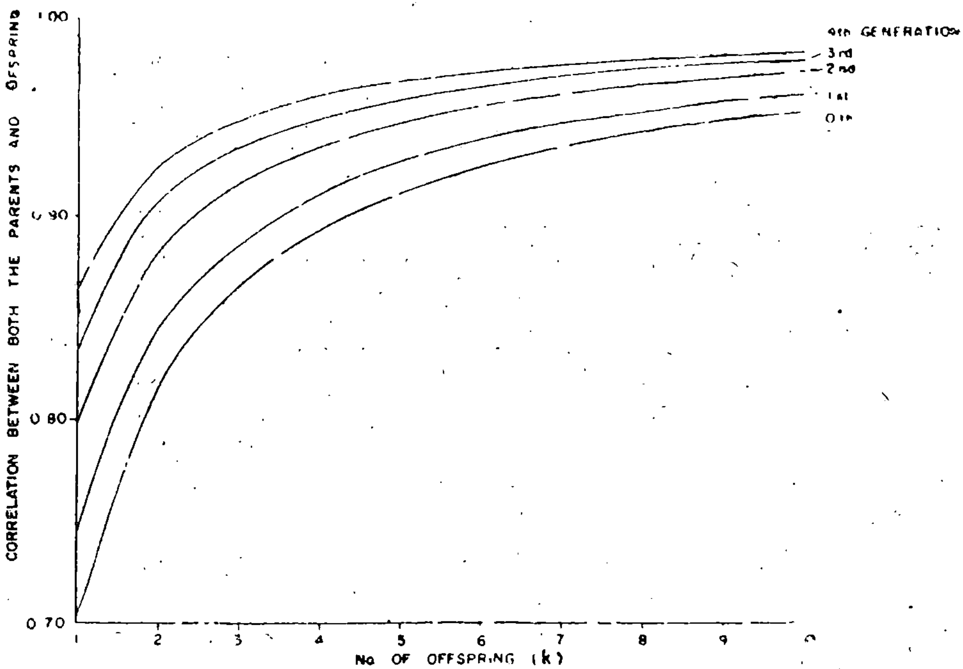

Fig. 1. Half-Sib Mating

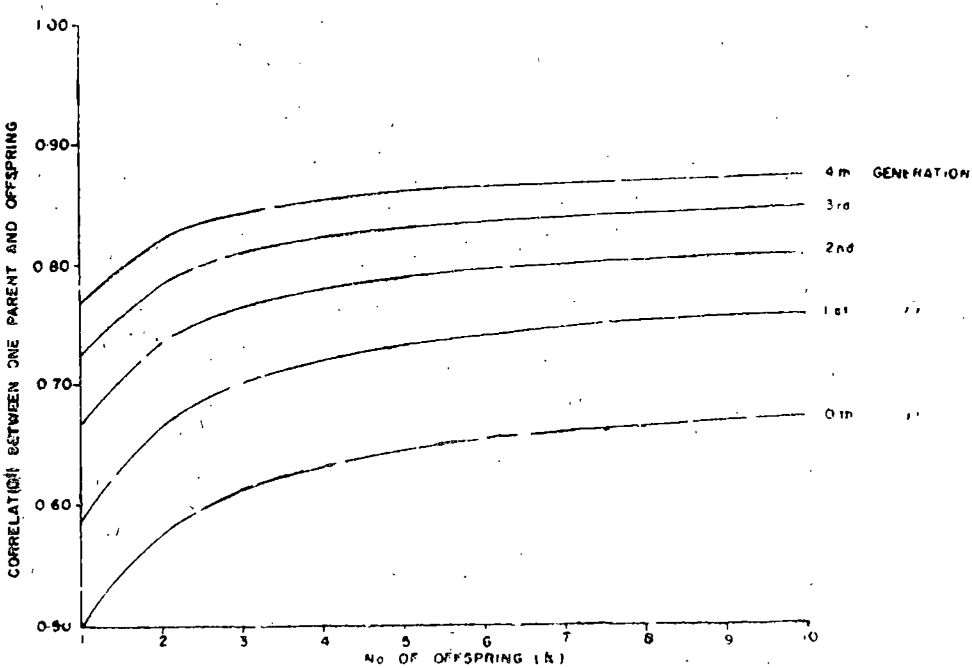

Fig. 2. Half-Sib Mating

TABLE 5

Correlation between one parent and K offspring when K=1 to 10 for the first four generation of half-sib mating.

Generation / *No. of offspring*	*0*	*1*	*2*	*3*	*4*
1.	0.5000	0.5892	0.6672	0.7248	0.7697
2.	0.5773	0.6681	0.7356	0.7861	0.8233
3.	0.6123	0.7024	0.7649	0.8102	0.8438
4.	0.6324	0.7216	0.7804	0.8231	0.8546
5.	0.6455	0.7340	0.7902	0-8312	0-8613
6.	0.6546	0.7426	0.7969	0.8367	0.8659
7.	0.6614	0.7489	0.8018	0.8407	0.8692
8.	0.6666	0.7537	0.8056	0.8437	0.8717
9.	0.6708	0.7576	0.8085	0.8461	0.8737
10.	0.6742	0.7607	0.8109	0.8480	0.8752

DISCUSSION

It is observed from the figures that the correlation increases as the number of offspring increases, but the rate of increase of the correlations were more in the case of both the parent and K offspring than that of one parent and K offspring case and the rate of increase is almost nil after the second generation of full-sib mating in the one parent case. The limiting value of the correlations in both the parent case tends to unity when K tends to infinity, where as in the one parent case the limiting values are $\sqrt{5/8}$, $\sqrt{25/36}$... etc for the lst 2nd etc......generations of half-sib mating when K tends to infinity. The correlations in both parent case are comparatively of higher order than that of one parent case. The correlation between one parent one offspring case canbe obtained when K=1. The results for random mating can be obtained from the generation corresponding to zero.

4. SUMMARY

A study of parent offspirng correlation under half sib mating system has been made in the case of autosomal genes. The correlation between both the parents and K offspring and between one parent and K offspring were worked out in the lines of George and Narain (1975) Tables of correlations of the above two cases when the number

of offspring varying between 1 to 10 were also worked out. A comprehensive study of these correlations graphically as well as numerically has been carried out.

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REFERENCES

[1] Fisher, R.A. (1918) : Correlation between relatives on the supposition of Mendelian inheritance: Trans. Roy. Soc. Edinb., 52. 399-433

[2] Fisher, R.A. (1949) : The theory of inbreeding ; 2nd Ed : 1965, Oliver and Boyd. London.

[3] George, K.C. (1974) : On correlation between relatives in inbred population Ph. D. thesis, submitted to I.A.R.I., New Delhi.

[4] George, K.C. and P. Narain (1975) : Parent-offspring correlations under full-sib and parent-offspring mating systems. *Jour. of Ind. soc. of Agri, Stat.* Vol XXVII, 50-70.

[5] Horner, T.W. (1956) : Parent-offspring and full-sip correlations under a parent-offspring mating system. *Genetics*, **41**, 430-468

[6] Kempthorne, O. (1955) : The theoretioalon values of correlation between relatives in random mating populations. *Genetics*, **40**, 153-167.

[7] Kempthorne, O. (1955c) : The correlation between relatives in inbred populations, *Genetics*, **40**, 681-691.

[8] Kempthorne, O. (1957) : An introduction to genetic statistics. *Join Wiley and Sons*, New York.

[9] Korde, V.T. (1960) : Correlation between relatives for a sex-linked character under inbreeding, *Herdity*, **14**, 401-409

[10] Li, C.C. (1955) : Population genetics, The university of Chicage press Limited : London

[11] Wright, S. (1921) : Systems of mating II. The effects of inbreeding on the genetic composition of a populaation. *Genetics*, **6**, 124-143.